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## New Constraints on Neutrino Oscillations in Vacuum as a Possible Solution of the Solar Neutrino Problem

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### Abstract

Two-neutrino oscillations in vacuum are studied as a possible solution of the solar neutrino problem. New constraints on the parameter  $\sin^2 2\theta$ , characterizing the mixing of the electron neutrino with another active or sterile neutrino, as well as on the mass-squared difference,  $\Delta m^2$ , of their massive neutrino components, are derived using the latest results from the four solar neutrino experiments. Oscillations into a sterile neutrino are ruled out at 99% C.L. by the observed mean event rates even if one includes the uncertainties of the standard solar model predictions in the analysis.

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Neutrino oscillations in vacuum [1], [2] have been discussed in connection with the solar neutrino experiments [3] and as a possible solution of the solar neutrino problem [4] for about 26 years [5]. In this scenario it is assumed that the state vector of the electron neutrino produced in the center of the Sun is a coherent superposition of the state vectors of neutrinos having definite but different masses. The flavor content of the neutrino state vector changes periodically between the Sun and the Earth due to the different time evolution of the vector's massive neutrino components. The neutrinos that are being detected in the solar neutrino detectors on Earth are thus in states representing, in general, certain superpositions of the states of  $\nu_e$ ,  $\nu_\mu$ ,  $\nu_\tau$ , and  $\nu_s$ , the latter being sterile neutrino. As the muon, tau and sterile neutrinos interact weaker with matter than electron neutrinos, the measured signals should be depleted with respect to the expected ones. This could explain the solar neutrino problem.

In order to make specific predictions for the signals in the solar neutrino detectors one should i) average over the neutrino production region in the Sun, ii) take into account the changing distance between the Earth and the Sun [6], and iii) integrate over the neutrino energy spectrum [7]. Qualitatively, the required depletion of the solar  $\nu_e$  flux can take place only if [5] the neutrino oscillation length in vacuum,  $L_v$ , is of the order of the distance between the Earth and the Sun,  $L_{se}$ : for  $L_v \gg L_{se}$  there will be no time for the oscillations to develop, and in the opposite case,  $L_v \ll L_{se}$ , the depletion is neutrino energy independent and can be at most  $1/N_f$ , where  $N_f$  is the number of weak eigenstate neutrinos taking part in the oscillations. Despite this "tuning" problem which has been addressed in several papers [8] and which is absent in the case of the MSW solution [9] of the solar neutrino problem, the vacuum oscillations provide an attractive explanation of the solar neutrino observations which should be further tested experimentally.

Analyses of solar neutrino data in terms of two-neutrino oscillations in vacuum have been made previously by several groups [10], [11], [12], [13]. It was found that a small region of values of the two parameters  $\Delta m^2$  and  $\sin^2 2\theta$  characterizing the oscillations,  $\Delta m^2 \cong (0.55 \div 1.1) \times 10^{-10} \text{ eV}^2$  and  $\sin^2 2\theta \cong (0.75 \div 1.0)$  [12], is allowed by the data. After the studies [10] – [13] were completed new data have been accumulated and published by three

of the four operating experiments.

In this letter the results of a joint analysis of the available data (including the latest results) from all solar neutrino experiments are presented. The first one is the pioneer Cl–Ar experiment by R. Davis and his group [14]. Data from 84 runs of measurements performed between 1971 and 1991 are available from this experiment. The first and the last day of the data taking period for each individual run have been taken from the table published in [15]. The second experiment is the one conducted by the Kamiokande collaborations which published recently their latest average results [16]. Data from 13 separate “runs”, taken between 1986 and 1990 are also available. Each “run” is three months long, only the last being slightly longer. The third experiment is the Ga–Ge experiment conducted by the SAGE collaboration at the Baksan Neutrino Laboratory. Only the last result [17] for the mean value of the  $^{71}\text{Ge}$  production rate has been used by us. Finally, the fourth experiment is the Ga–Ge one conducted by the GALLEX collaboration, from which new data from 30 accomplished runs have become available recently [18].

The data analysis has been performed in two different ways. First, we perform a  $\chi^2$ –analysis of the mean values of the event rates in each of the four detectors. We compare the event rates expected, assuming neutrino oscillations in vacuum take place between the Sun and the Earth, with the experimentally measured event rates. The expected event rates without oscillations, as well as the spectra of the different components (pp,  $^7\text{Be}$ ,  $^8\text{B}$ , etc.) of the solar neutrino flux have been taken from [19]. For the ratios of measured to expected event rates in each solar neutrino detector the following mean values and their corresponding error bars have been used:

$$R_{Cl} = 0.29 \pm 0.03, \tag{1}$$

$$R_{\text{SAGE}} = 0.53 \pm 0.19, \tag{2}$$

$$R_{\text{GALLEX}} = 0.60 \pm 0.09, \tag{3}$$

$$R_{\text{Kamioka}} = 0.51 \pm 0.07. \quad (4)$$

The errors in (1) – (4) are the added in quadrature statistical and systematic errors as separately estimated by each collaboration.

It has been argued [19], [20], [21] that the theoretical uncertainties of the standard solar model alone cannot account for the discrepancy between theoretical predictions and the experimental results. However, these uncertainties have to be taken into account in a conservative analysis of the data, as in some cases they are bigger than the experimental errors. We include in our analysis the theoretical uncertainties, as well as the correlations between the uncertainties in the predicted event rates in the different solar neutrino detectors, as described in [22] for an analogous MSW analysis. The uncertainties of the different solar neutrino fluxes and detection cross-sections estimated in [19] have been used. We include an uncertainty of two percents for the low energy electron–neutrino scattering cross-section which has not been measured with high enough precision at the neutrino energies of interest. Finally, we prefer to treat the SAGE and GALLEX results as independent measurements and do not use the corresponding weighted average result.

The probability for an  $\nu_e$  to remain  $\nu_e$  has been averaged over a period of one year taking into account the ellipticity of the Earth orbit, as described in [12]. Both the change of the survival probability and the change of the total flux with the distance between the Sun and the Earth have been included in the calculation.

The comparison between expected and measured event rates in each detector has been made for a sufficiently large number of pairs  $\Delta m^2$  and  $\sin^2 2\theta$ . For oscillations into active neutrinos,  $\nu_e \leftrightarrow \nu_a$ ,  $\nu_a$  being either  $\nu_\mu$  or  $\nu_\tau$ , the minimal  $\chi^2$  ( $\chi^2_{\min}$ ) is 4.6 with theoretical uncertainties included in the analysis, and 5.2 if the theoretical uncertainties are neglected. With four experimental results (two degrees of freedom) this means that as a solution of the solar neutrino problem the  $\nu_e \leftrightarrow \nu_a$  oscillations are ruled out at 90% C.L., but are allowed at 95% C.L. Accepting the hypothesis that the  $\nu_e \leftrightarrow \nu_a$  oscillations provide the solution of the solar neutrino problem, the 90% C.L. and 95% C.L. allowed regions of values of  $\Delta m^2$

and  $\sin^2 2\theta$  are shown in Figs. 1. The results depicted in Fig. 1a have been obtained with the theoretical uncertainties taken into account as discussed earlier, while the results shown in Fig. 1b have been derived without including the theoretical uncertainties in the analysis.

For solar neutrino oscillations into a sterile neutrino,  $\nu_e \leftrightarrow \nu_s$ , the same analysis shows that  $\chi^2_{min} = 10.1$  and  $\chi^2_{min} = 11.1$ , respectively. Thus, the  $\nu_e \leftrightarrow \nu_s$  oscillations are ruled out at 99% C.L. as a solution of the solar neutrino problem \*. In case one accepts to combine the results of the two Ga-Ge detectors and to have only three data points, the oscillations into sterile neutrinos are excluded at 99.5% C.L. †

Let us note that in none of the previous studies [10] - [13] the constraints on  $\Delta m^2$  and  $\sin^2 2\theta$  following from the mean event rate data only were derived.

A second approach to the analysis of the solar neutrino data has been proposed and described in detail in [12]. It is more suited for an analysis of data that is supposed to vary periodically with time. In such case the time intervals over which the detected signal is averaged should be chosen shorter than the period of the anticipated variations in order not to smear out the latter. Therefore, for neutrino oscillations in vacuum data averaged over shorter than one year periods should be used and the comparison should be made on a “run by run” basis. This method allows to rule out certain regions of parameters that are allowed by the analysis which makes use only of the mean values of the measured event rates. Although the individual runs have large error bars, the effect of a systematic discrepancy between predicted and observed event rates results in a higher  $\chi^2$  for certain values of  $\Delta m^2$  and  $\sin^2 2\theta$ . On the other hand, the opposite effect might also occur. When

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\*This result depends slightly on the threshold used in the Kamiokande detector, which here was assumed to be 7.5 MeV.

†Let us note that the MSW  $\nu_e$  transitions into a sterile neutrino  $\nu_s$  also give a rather poor fit of the mean event rate data: in this case  $\chi^2_{min} = 3.43$  which implies that the indicated transitions are ruled out as a solution of the solar neutrino problem at 80 % C.L. [23].

introducing more degrees of freedom the overall  $\chi^2$  might increase slower than the value of the percentage point,  $\chi_p$ , for the corresponding number of degrees of freedom. Therefore values of the parameters ruled out by the analysis using mean values only, might become allowed if one utilizes the “run by run” data.

For the analysis of the data from the solar neutrino experiments in terms of neutrino oscillations in vacuum this approach was first pursued in [12] using only the results of the Homestake and Kamiokande–II experiments. Here the same procedure is applied adding the additional information about the measured  $^{71}\text{Ge}$  production rate in each of the 30 individual runs completed by the GALLEX collaboration. In the case of vacuum neutrino oscillations the variations of the event rates in any solar neutrino detector resulting from the change of distance between the Sun and the Earth (apart from the standard geometrical one) are mostly due to  $^7\text{Be}$  neutrinos. The latter have a very narrow spectrum [24] which can be approximated by a line. Therefore the averaging over the continuous spectrum of the other components of the solar neutrino flux leads to much less pronounced variations of the signals due to these neutrinos as compared with the signals due to  $^7\text{Be}$  neutrinos [12], [23] (see also [7]). In the Homestake detector the  $^7\text{Be}$  neutrinos contribute only about 14 % of the total expected signal, whereas in the Ga–Ge detectors they contribute  $\sim 25$  % of the signal. Therefore, if vacuum neutrino oscillations are the solution of the solar neutrino problem, the seasonal variations of the signal in Ga–Ge detectors should be somewhat stronger than in Cl–Ar ones. As the GALLEX data seems to be rather constant with time, one expects that certain regions of  $\Delta m^2$  and  $\sin^2 2\theta$  parameters can be ruled out. However, the results obtained within the more detailed approach cannot be directly compared with those presented in Figs. 1 as data for short time intervals from the Kamiokande experiment for the period since 1990 have not been published yet. In order to compare the effectiveness of the two approaches we give below also the constraints following from the mean values of the measured to expected event rates for the Homestake experiment, eq. (1), the GALLEX, eq. (3), and the Kamiokande–II experiment

$$R_{\text{Kamioka}} = 0.46 \pm 0.08. \quad (5)$$

Note also, that we have not included the theoretical uncertainties in the "run by run" analysis because of the formidable computational difficulties of inverting large correlation matrices.

Our results for the case of  $\nu_e \leftrightarrow \nu_a$  oscillations are shown in Figs. 2. When only the mean values are taken into account one obtains  $\chi^2_{\min} = 2.8$ . For 3 data points (1 degree of freedom) this implies that the hypothesis of  $\nu_e \leftrightarrow \nu_a$  oscillations being the solution of the solar neutrino problem is ruled out at 90% C.L. but cannot be ruled out at 95 % C.L. Once this hypothesis has been accepted, the regions of  $\sin^2 2\theta$  and  $\Delta m^2$  allowed at 90% C.L. and at 95% C.L. in this case are shown in Fig. 2a.

With the run by run event rates used in the analysis we have  $\chi^2_{\min} = 117$  for 125 degrees of freedom, which implies a good quality of the fit. The allowed regions of parameters  $\Delta m^2$  and  $\sin^2 2\theta$  at 90% C.L. and 95% C.L. are shown in Fig. 2b. They are considerably narrower than those depicted in Fig. 2a. The region around  $\Delta m^2 = 1.1 \times 10^{-10} \text{ eV}^2$ , which is allowed at 90% C.L. if one uses the mean values in the analysis, is practically completely ruled out even at 95 % C.L. by the run-by-run data.

The same analysis has been performed for the hypothesis of solar neutrino oscillations into a sterile neutrino,  $\nu_e \leftrightarrow \nu_s$ . With only the mean values used in the analysis the minimal  $\chi^2$  is equal to 6.7. Consequently, the  $\nu_e \leftrightarrow \nu_s$  oscillations give a poor fit of the mean event rate data (1), (3) and (5): they are marginally allowed only at 99.5% C.L. The situation is quite different when the variations are taken into account. In this case  $\chi^2_{\min} = 123$ , which means that the  $\nu_e \leftrightarrow \nu_s$  oscillations are now allowed even at 68% C.L. The corresponding 90% C.L. and 95% C.L. allowed regions of  $\Delta m^2$  and  $\sin^2 2\theta$  are shown in Fig. 3. It should be noted that the latest GALLEX-I results not only do not constrain further the allowed values of the parameters  $\Delta m^2$  and  $\sin^2 2\theta$ , but actually slightly relaxes the constraints obtained from the Homestake and Kamiokande-II data only. Thus, we see that the mean event rate data are much more restrictive in the case of vacuum oscillations into sterile neutrino (to

the point of practically excluding them as a possible solution of the solar neutrino problem), than the run-by-run data. The inverse is true for the  $\nu_e \leftrightarrow \nu_{\mu,\tau}$  oscillations: the stronger restrictions on the parameters follow from the run-by-run results.

In conclusion, the solution of the solar neutrino problem in terms of two-neutrino oscillations in vacuum has been confronted with the latest data from all solar neutrino experiments. It has been shown that previously allowed regions of the parameters  $\sin^2 2\theta$  and  $\Delta m^2$  are ruled out by the data from individual runs of the Homestake and GALLEX experiments, and data from the Kamiokande-II detector, averaged over three-month periods. The  $\chi^2$ -analysis of the current mean event rates in the Homestake, Kamiokande, SAGE and GALLEX detectors based on the Bahcall – Pinsonneault theoretical predictions rules out two-neutrino  $\nu_e$  oscillations into sterile states as a solution of the solar neutrino problem at 99% C.L., with the uncertainties in the theoretical predictions included in the analysis.

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**Note Added.** The present Updated Version of our preprint IFP - 480 - UNC, Ref. SISSA 177/93/EP, differs from the one published in November 1993 by the data from the GALLEX collaboration used as input in the analyses: here we utilize the latest GALLEX results published in February of 1994, which are based on data from 30 completed runs of measurements (see ref. [12]); in the version of our work from November 1993 only the data from the first 15 accomplished runs of the GALLEX experiment, available at that time, were used. As a consequence, the results presented here differ somewhat from the results published in November 1993.



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## Figure Captions

**Fig. 1** Regions of values of the parameters  $\Delta m^2$  and  $\sin^2 2\theta$  allowed at 90 % C.L. (solid line) and 95 % C.L. (dashed line) in the case of vacuum  $\nu_e \leftrightarrow \nu_{\mu(\tau)}$  oscillations of solar neutrinos. The mean event rates measured by the Homestake, Kamoikande, SAGE and GALLEX collaborations, and the SSM predictions of Bahcall and Pinsonneault have been used in the  $\chi^2$ -analysis. The results shown have been obtained a) by including, and b) without including, the theoretical uncertainties in the analysis.

**Fig. 2** Regions of values of the solar neutrino  $\nu_e \leftrightarrow \nu_{\mu(\tau)}$  oscillation parameters  $\Delta m^2$  and  $\sin^2 2\theta$  allowed at 90 % C.L. (solid line) and 95 % C.L. (dashed line). Fig. 2a has been obtained by using only mean event rates (eqs. (1), (3), and (5), see the text). The results of the extended  $\chi^2$ -analysis based on 84 runs of the Homestake experiment, 30 runs of the GALLEX experiment and 13 three-month time-intervals of the Kamiokande-II experiment are given in Fig. 2b.

**Fig. 3** The same as in Fig. 2b for solar neutrino oscillations into sterile neutrino in vacuum:  $\nu_e \leftrightarrow \nu_s$ . The black dots (seen at  $\Delta m^2 \cong 7.9 \times 10^{-11} \text{ eV}^2$  and  $\sin^2 2\theta \cong 0.79$ ) correspond to values of  $\Delta m^2$  and  $\sin^2 2\theta$  allowed at 95% C.L.; they also give an idea about the precision of the calculations.

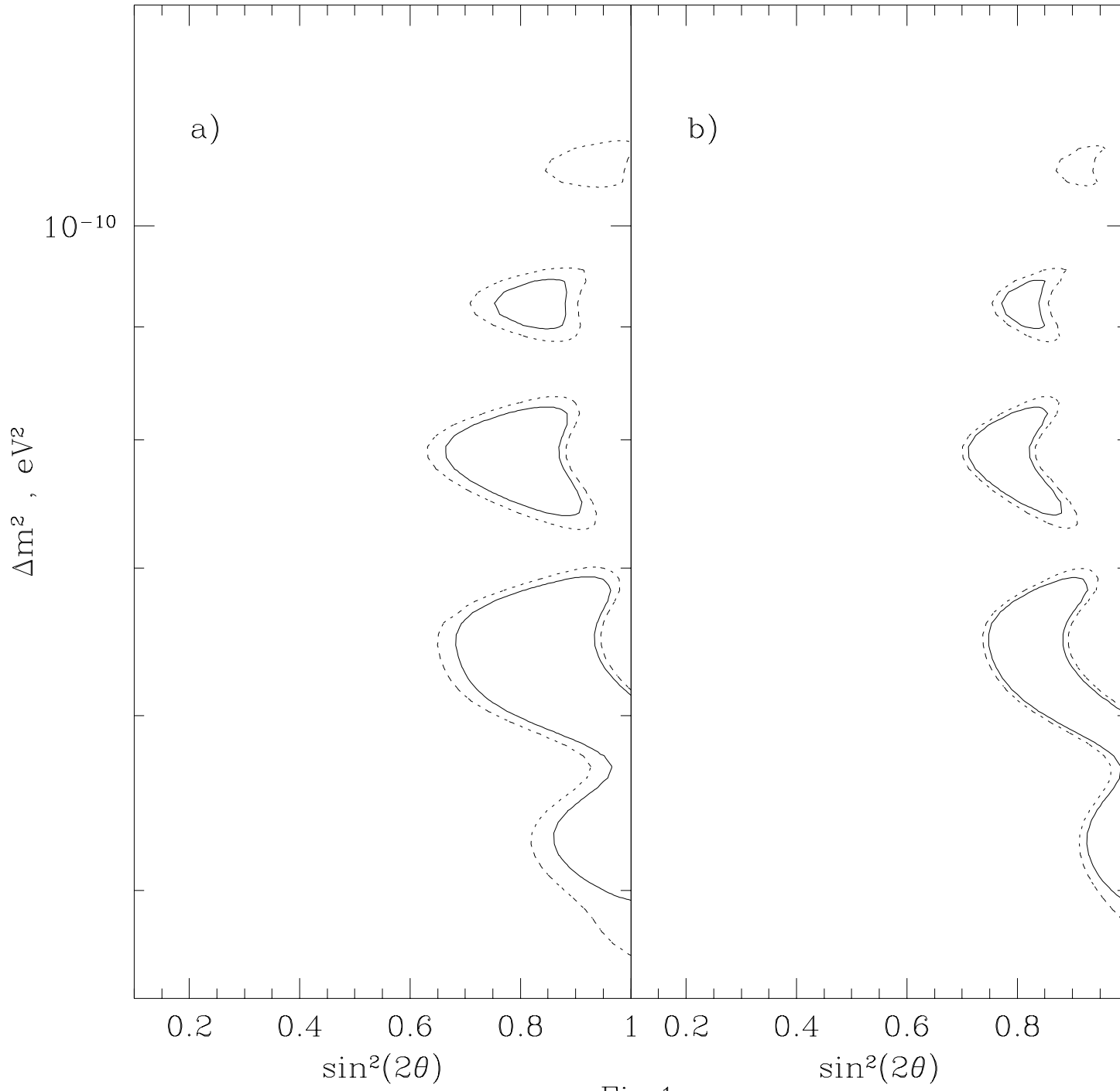


Fig.1

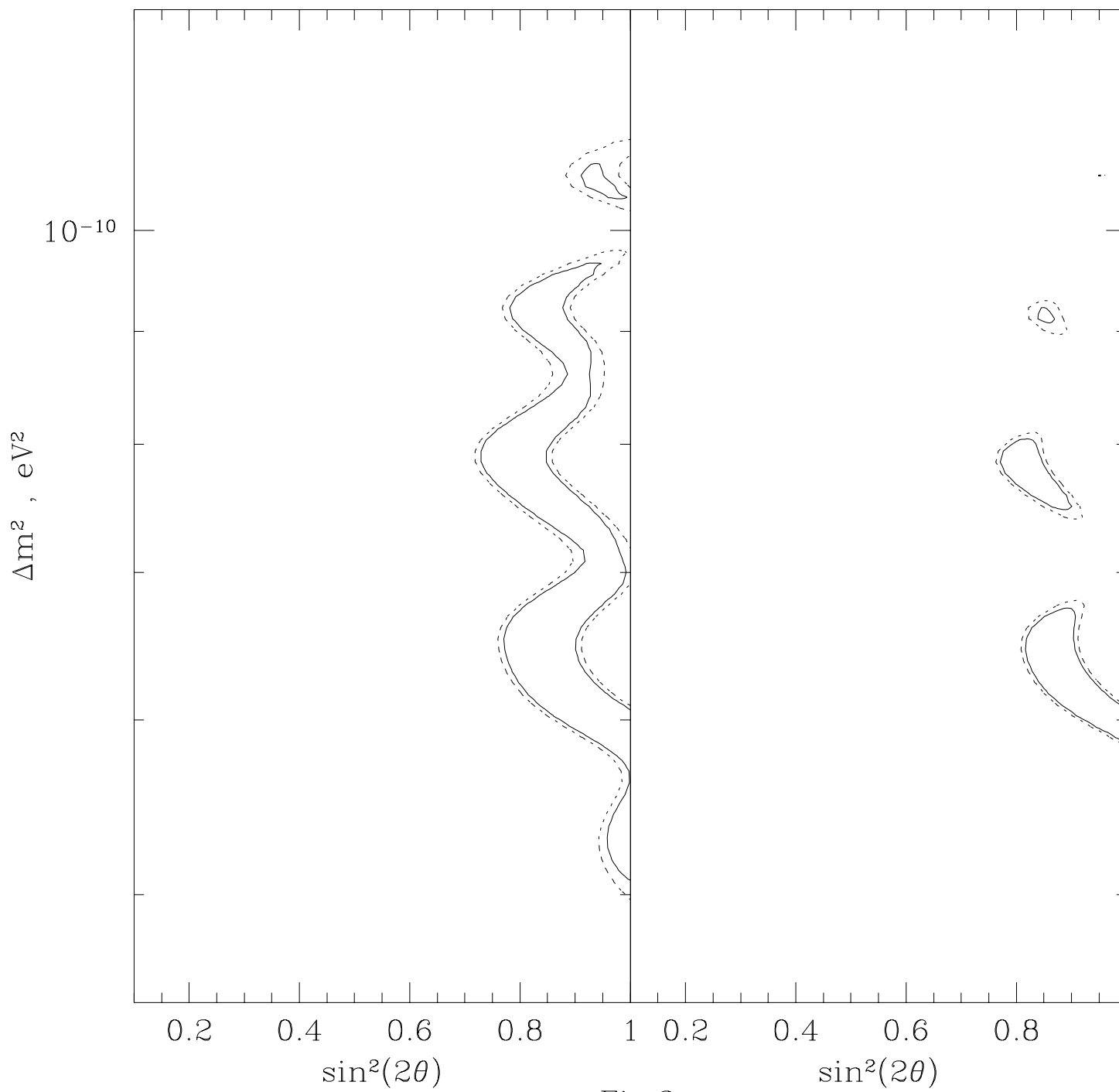


Fig.2

